MAE 4540 Final Project Report: Investigating the Trade-off Between Operating Altitude vs. Drag Compensation for Maximizing Payload Mass of a Spacecraft

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Abstract (Mohammad)

The overall objective of this project was to investigate the trade-off between operating altitude and drag compensation. This optimal operating altitude at low-Earth orbit maximizes Falcon 9's payload volume. Based on computation in MATLAB verified with simulation in GMAT, an altitude on the order of $300km$ was suitable for maximizing the payload mass to be on the order of $9000kg$. The desired mission duration and payload volume was estimated to be 5 years based on the application of an in-space manufacturing depot from research and discussions with the founder of Varda Space Industries. Research conducted on optimizing the spacecraft's shape for low drag while maximizing volume resulted in a drag coefficient of 2.72. By investigating different propulsion systems' performance parameters, the ion engine, QinetiQ T5, was selected as the best propulsion system for this missions because it offers an I_{sp} of 3000 seconds while having a light mass of only $50kg$.

1 Introduction (Matt)

With an increasing need for complex payloads and longer duration missions, companies are looking for solutions to increase the efficiency of their spacecraft. While some space missions are far enough away from earth that drag is a negligible factor, those that require operation within Low or Very Low Earth orbit need to be optimized to operate successfully in these conditions. One such mission is Varda's space factory facility that will operate in LEO due to the need for a micro-gravity environment. The purpose of this report will be to explore the trade-offs between operating altitude and drag compensation. The end goal will be to determine the optimal altitude for the Falcon 9 that allows for the maximum payload. This will be accomplished by utilizing a MATLAB analysis that will determine the optimal altitude for maximum payload that utilizes the mission duration, spacecraft drag and specifications of propulsion systems as inputs. This analysis will then be further verified in GMAT.

1.1 Industry Partner (Kenny)

Our industry contact for this project was Will Bruey, founder of Varda Space Systems. Varda Space Systems is a startup with the vision of making in-space manufacturing mainstream. They envision robotic mini space stations in low earth orbit producing high-margin specialty goods such as high-purity optical fiber and novel pharmaceuticals that would benefit from a zero-g environment. Mini uncrewed capsules would shuttle raw materials up to these manufacturing stations and return the finished goods to Earth to sell. The premise of our project is exactly the design study Varda is performing for the design of their manufacturing space station.

We had email correspondence and a thirty-minute in-person meeting with Will Bruey during the course of our project. These are the key points we were able to learn about Varda's mission as it pertained to our trade study:

- The space station will be comprised of two modules, launched together. One is an approximately 1 ton (1000 kg) common satellite bus (containing propulsion, power, flight computers, attitude determination and control, etc...) while the second is a configurable module containing all the manufacturing hardware.
- The minimum life span of the station is 5 years.
- The manufacturing activities will consume about 2kW of power while underway.
- The station should utilize as much commercially available hardware as possible. In-house designs are to be kept at a minimum with their current business model.

1.2 Trade-off Parameters (Mohammad)

To carry the maximum payload mass to LEO, external forces that negatively affect the performance and orientation of a spacecraft need to be analyzed. The force of drag induced by the dense atmosphere at Earth is one important example of these external forces. The equation of drag force can be described as

$$F_d = \frac{1}{2}\rho v^2 C_d A, \tag{1}$$

where ρ is density of the fluid (air in this study), v is the velocity of the spacecraft relative to the fluid, C_d is the coefficient of drag, and A is the cross sectional area of the spacecraft.

Intuitively, at lower operating altitudes, it is easier and cheaper to propel a spacecraft and place into orbit, than at higher altitudes. This is because less propellant is needed due to lower displacement between launch site and orbit altitude, allowing room for increase in the payload mass. On the other hand, due to lower density at higher altitudes, less thrust and use of propellant is needed in order to operate in an orbit. In this trade-off study, three parameters were investigated to maximize the payload mass of a spacecraft launch mission to operate at low-Earth Orbit (LEO): the shape of the spacecraft affecting the cross-sectional area and coefficient of drag, the propulsion system, and the density as a function of altitude on Earth.

2 Analysis of Trade-off Parameters

2.1 Altitude and Air Density (Alex Mohammad)

Multiple atmospheric models were considered to understand the behavior of density as the altitude increases on Earth. Choosing the correct atmospheric model and assumptions is vital for this project; atmospheric density is the key parameter that changes with altitude in the drag force equation. As noted by McLaughlin et al. (2011), "Atmospheric density modeling is the greatest uncertainty in the dynamics of low-Earth-orbit satellites." [12]. For Drag analysis in low Earth Orbit, the Jacchia-Roberts Empirical atmosphere is often used [13]. Unlike the U.S. Standard Atmosphere, which provides a range of densities based on altitude and loses accuracy above the Von Karman Line, Jacchia-Roberts takes in account position and time, as solar flux can significantly impact atmospheric density at high altitudes [13]. These changes are readily apparent in the Drag parameter analysis for the GRACE mission, a LEO satellite that consistently used a propulsion system to maintain net zero drag. The GRACE Mission used a circular LEO orbit and a constant spacecraft profile, so all changes in drag were caused by fluctuations in density.

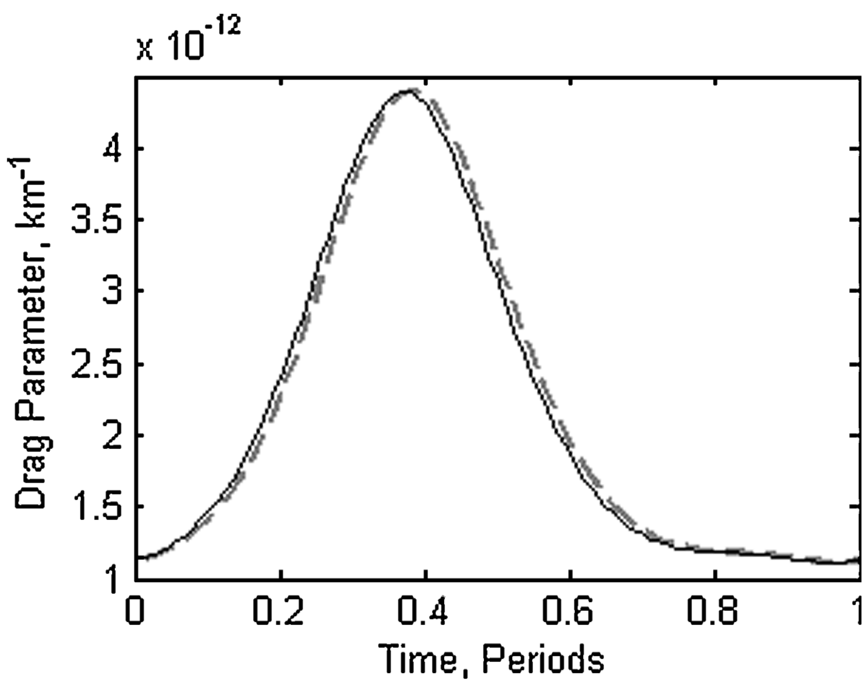


Figure 1: Analysis of Atmospheric Drag in the GRACE Mission [13]

As evident by this plot, atmospheric drag is cyclic over each orbital period. To create the most accurate drag modeling, an analysis must require constant propagation. Propagation on this scale is outside the initial scope of this paper; our analysis will instead aim for a a rough first order approximation of drag by assuming a constant density at a given altitude. Because the drag parameter is cyclic, we can approximate the mean value per rotation as time invariant.

Our analysis uses an Open Source Jacchia-Roberts MATLAB Library originally developed by David Eagle in 2021. [14]. We assumed a launch date of October 1, 2023 and a Latitude of 28.5°, the position of Kennedy Space Center, the launch site.

Figure 2 illustrates the resultant densities with respect to altitude. Our results agree with other atmospheric models in previous studies [1, 2].

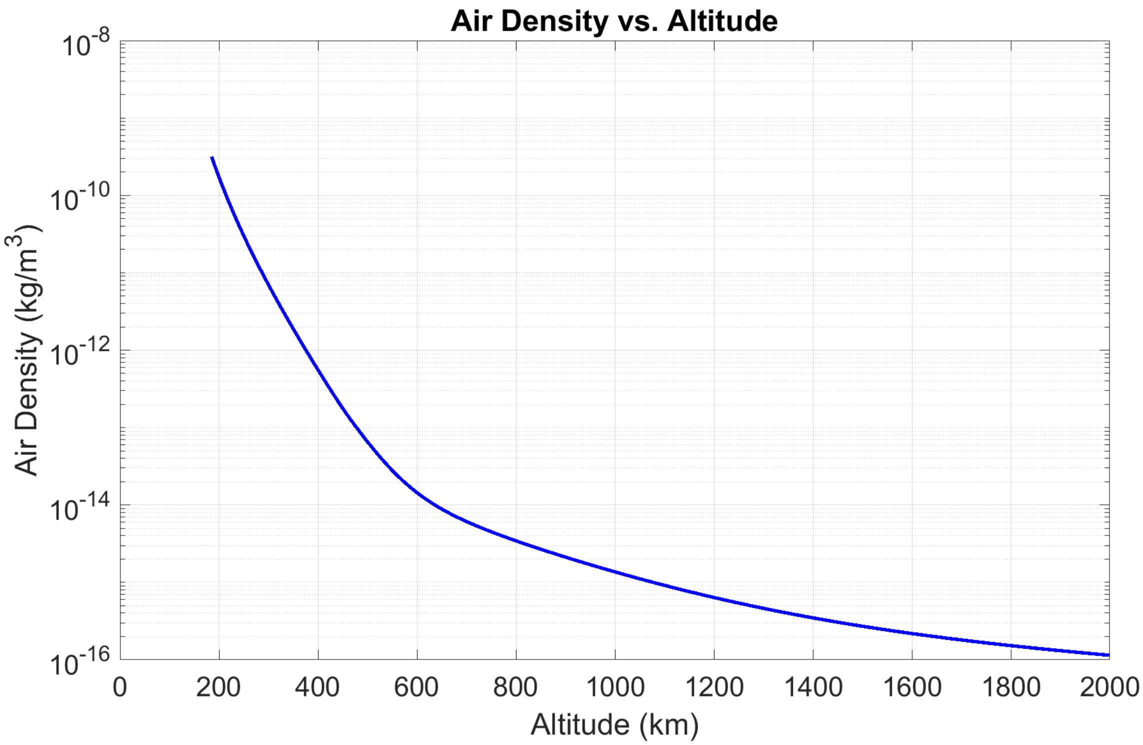


Figure 2: Atmospheric Density vs Altitude based on custom Jacchia-Roberts Implementation [14]

2.2 Drag Reduction (Matt)

Reducing the amount of drag produce by a spacecraft is vital to ensure efficient operations and reducing the amount of energy and fuel expending during flight. A computational analysis study by Walsh and Berthoud investigated the effects of shape variations and their effect on drag and internal payload volume. The study demonstrated a few key points in spacecraft shape design. The first aspect that was analyzed was the nose geometry. As seen in Figure 3 increasing the cone angle reducing the drag coefficient.

Fixed Cone Length (m)	Optimum Body for Minimum Drag	C_D (% reduction on no cone)	Volume [m^3]
No Nose Cone		2.72	1.00
0.1		2.62 (3.5%)	0.99 (1%)
0.2		2.55 (6.3%)	0.96 (4%)
0.3		2.47 (9.1%)	0.93 (7%)

Figure 3: Comparison of Optimal Nose Geometries for minimum drag on 2m body [3]

Figure 3 demonstrates that with no nose cone, one has 9% higher drag coefficient when compared to cone nose with a length of 0.3 m in the context of the study. In addition, it was shown that reducing the cone radius to a minimum, or ideally zero, allows for a lower level of drag and should be the goal. In addition, an increase in cone length and angle does not have that large of an effect on volume reducing with the study only showing a 7% decrease in payload volume. Another key factor to consider is the tail geometry. As seen in Figure 4, reducing the tail radius has a direct correlation to reduction in drag.

Fixed Tail Radius (m)	Optimum Body for Minimum Drag	C_D	Volume [m^3]
		(% reduction on no cone)	
No Nose Cone		2.72	1.00
0.20		2.54 (6%)	0.92 (8%)
0.15		2.44 (10%)	0.85 (15%)
0.10		2.39 (12%)	0.78 (22%)

Figure 4: Comparison of Optimal Tail Geometries for minimum drag on 2m body [3]

However, Figure 4 also shows that the internal volume change associated with reducing the tail radius could be considered an unacceptable level at 22% when the radius is reducing to 0.1 m in this study.

This demonstrates that there is a specific balancing act between drag reduction and maintaining internal volume space. The best approach to this optimization in the context of pure drag reduction through the drag coefficient would be to identify the necessary payload for the mission then optimizing the nose and tail geometry. This ensures that one can maintain a sufficient amount of internal volume while having the lowest amount of drag possible under the conditions.

For our analysis, we assumed three spacecraft configurations and evaluated their drag coefficients and drag areas accordingly as detailed in Table 1. For all of the cases we assumed that the drag area of the spacecraft chassis was the cross-sectional area of the Falcon 9's 4.5m fairing. This is because Varda intends to maximize the volume of payload they can fit into the Falcon 9. Solar cells are assumed to be body-mounted on the spacecraft except one of the configurations which adds an additional 20 m^2 of solar array wings. $20m^2$ will comfortably generate the 2kW of power needed for manufacturing operations required by Varda even at spacecraft end of life assuming a $1000W/m^2$ power density. The C_d=2.72 configurations correspond to a flat leading and trailing face while the C_d=2.39 design has an aggressive tapered tail as described earlier in this section. Figure 5 is a plot of the calculated drag force for the C_d=2.72 configuration with no solar array wings as a function of altitude:

Table 1: Drag Parameters

Configuration	C_d	**Drag Area,** m^2
Flat face, body solar cells	2.72	16.42
Tapered tail, body solar cells	2.39	16.42
Flat face, solar array wings	2.72	36.42

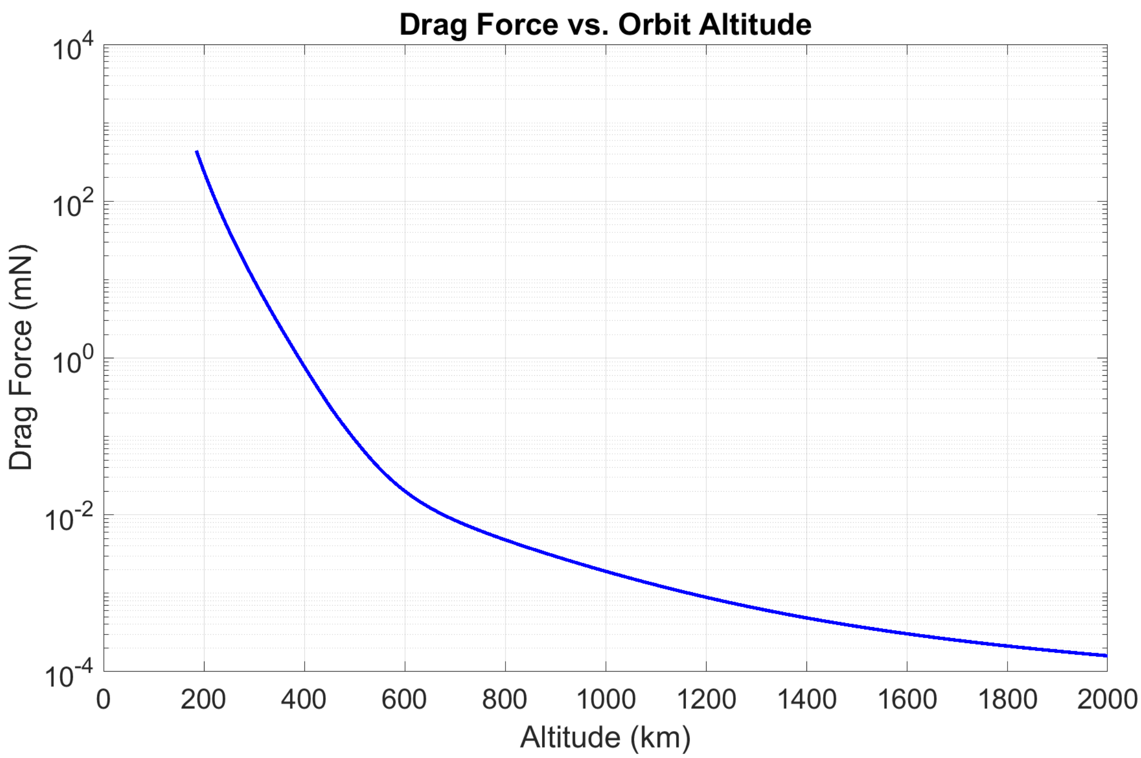


Figure 5: Drag vs. Orbit Altitude for C_d=2.72 and drag area=16.42m^2

2.3 Payload Capacity (Alex)

Due to its low cost per launch, reliability, and relative flexibility working with commercial partners, a Falcon 9 rocket configured for reusable missions was selected as a launch vehicle for this mission. Because the exact payload capacities are proprietary and can vary based on mission, there is very limited available information on Payload Capabilities for the most up-to-date Falcon 9 configurations [15]. Most Falcon 9 User Guides from the past decade include a message to contact SpaceX directly for additional information on payload capabilities. However, User Guides from 2013 and earlier have more complete information. Although the Falcon 9 performance has improved considerably since this time, we will use these older editions for a first-order approximation.

The 2009 Falcon 9 Users Guide provides a list of its payload capacities to different orbital inclinations based on altitude [16]. Based on these charts, circular Low Earth Orbits with an inclination of 28° have the optimal payload capabilities. The chart lists a set of points ranging from 200 to 2000 km. In addition to this range, we wanted to test an additional datapoint closer to the Von Karman line. For this, we used a 2017 NASA report which set the payload at 185 km to 13.15 t. [17] Adding this to the datapoints from the user guide, we can use linear interpolation to return payload mass as a function of altitude. Figure 6 shows the resultant graph. This linear interpolation strategy is an estimate: future approaches could use contemporary SpaceX data to provide more accurate predictions.

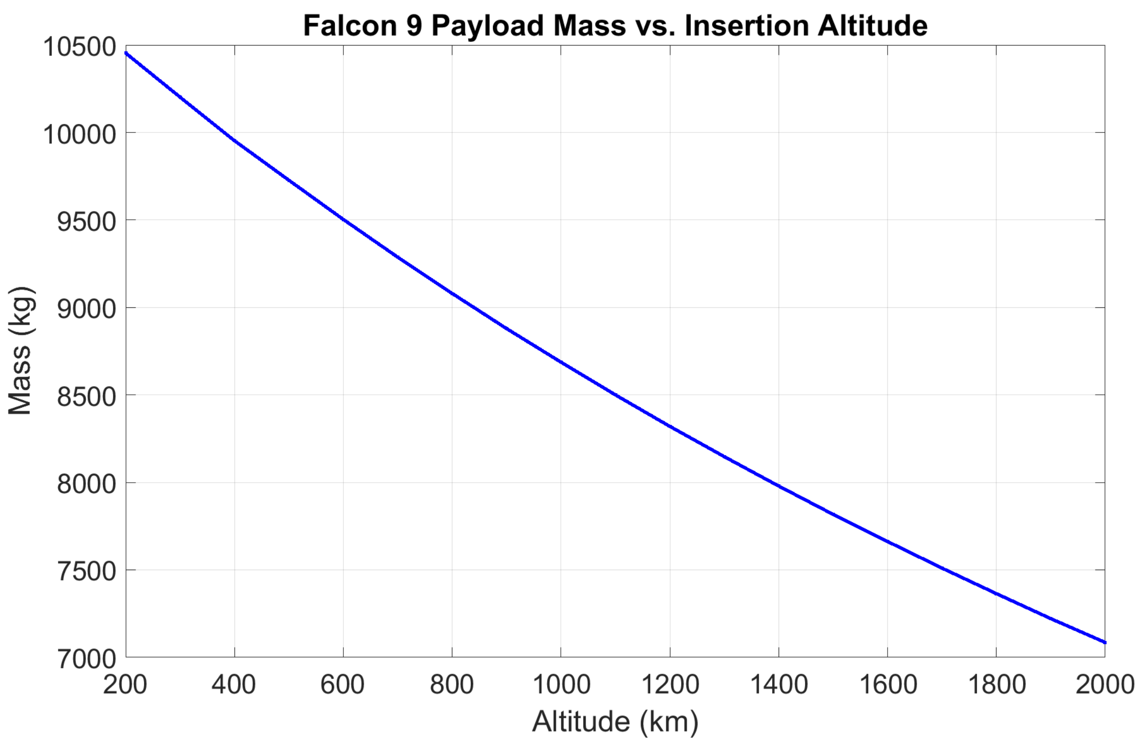


Figure 6: Payload Capabilities for a Falcon 9 Rocket at 28° inclination with variable orbit altitudes

2.4 Propulsion System (Kenny)

From the analysis presented in the prior two sections and our discussions with Mr. Bruey from Varda, we set following propulsion system requirements:

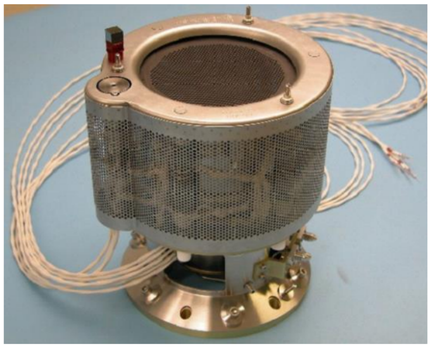
- The propulsion system shall provide orbit maintenance and attitude control. No orbital maneuvering or orbit raising capability is required (Falcon 9 will preform a direct injection).
- The system must operate for a minimum of 5 years.
- Thrust on the millinewton order of magnitude is required for drag compensation per the drag force analysis in a prior section.

These requirements point clearly in the direction of an electric propulsion system for orbit maintenance. A variety of EP systems we discussed in class could provide this millinewton order of magnitude of thrust we require with very high specific impulse when compared to chemical systems. Especially with such a long mission life, a high specific impulse is critical for reducing required propellant mass. An additional benefit of electric propulsion is that it generates significantly less vibration and shock then chemical systems, allowing sensitive on-board manufacturing to continue even during burns. Also, with over 2kW of power needed on-board for manufacturing anyway, there should be minimal additional hardware needed to power an EP system. For the same reason, the attitude control system should be comprised of reaction wheels or control moment gyroscopes instead of chemical thrusters. Either would be able to synergize with the EP power supply and would not require depleting propellant.

In our conversations, Mr. Bruey was very clear that the success of Varda's business model is predicated on heavy utilization of commercial off-the-shelf (COTS) hardware. As such, we limited our analysis to existing COTS hardware with flight heritage instead of specifying parameters for a custom design. The QinetiQ T5, SPT-100, and L3 XIPS-25 systems were considered and their parameters are outlined in Table 2:

Table 2: Propulsion System Parameters [4] [5] [6] [7]

EP System	**EP Type**	I_{sp}	**Max Thrust**	**Dry Mass (est.)**
QinetiQ T5	Ion Engine	3000 sec	20 mN	50 kg
Russian SPT-100	Hall Effect	2200 sec	80 mN	100 kg
L3 XIPS-25	Hall Effect	3400 sec	165 mN	100 kg



(a)

(b)

Figure 7: QinetiQ T5 Ion Engine [4](a) and SPT-100 Hall-Effect Thruster [6](b)

In researching the three systems, the flight heritage of the QinetiQ T5 was of particular interest. The T5 powered a 2009 ESA mission called GOCE which operated at a very low altitude of just 255km which was required for its mission of mapping surface magnetic fields. Much like our intended CONOPS, GOCE used it's T5 ion engine almost continuously for drag compensation, expending 40kg of Xenon propellant over its two year life [5]. As a follow on to the success of the T5, ESA is researching air-breathing ion engines for future very low altitude missions. The concept is to develop an ion engine that would intake the atmospheric air to use as an "in-situ" propellant, greatly increasing possible mission length. Such a technology is likely still decades away from maturity, but it offers an exciting long-term solution for applications such as ours.

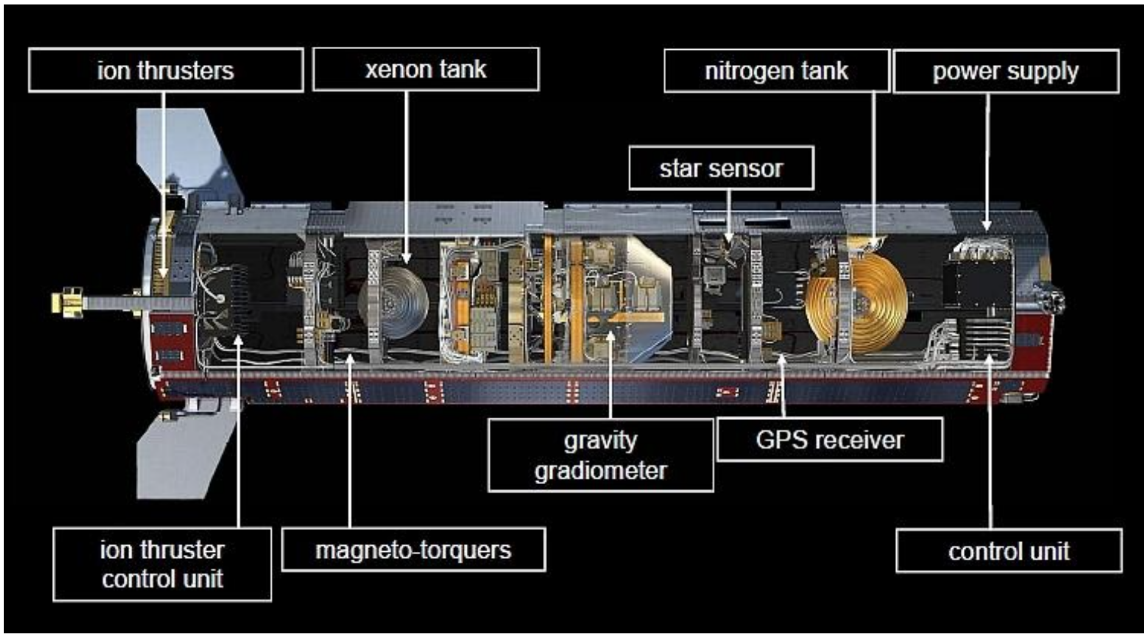


Figure 8: The GOCE spacecraft, featuring two redundant T5 ion engines, drag-minimizing shape, and stabilizing fins. Image Source: ESA

3 MATLAB Analysis (Kenny and Alex)

A MATLAB script was written to synthesize all of the trade-off parameters detailed in the previous section and calculate an optimal operating altitude for the Varda manufacturing space station. The code is structured around three nested loops. The first loops through the three different propulsion systems, and the second through the three drag configurations. The inner loop loops through all possible altitudes, defined as between 200 to 2000 km (the edge of LEO). At each altitude, the air density is calculated per the Jacchia model as detailed before.

Then, drag is calculated per Equation 1 as detailed above with the velocity of the circular orbit being calculated with Equation 2 where d is the orbital altitude:

$$v = \sqrt{\frac{Gm_{earth}}{r_{earth} + d}} \tag{2}$$

The thrust from the propulsion system for drag compensation is assumed to be equal and opposite the drag force for continuous, steady-state flight per Equation 3. Mass flow rate and then propellant mass is then calculated with Equations 4 and 5 where $t_{mission}$ is the mission life of 5 years:

$$\sum F = 0 = F_{thrust} - F_{drag} \tag{3}$$

$$\dot{m} = \frac{F_{thrust}}{I_{sp}g} \tag{4}$$

$$m_{prop} = t_{mission}\dot{m} \tag{5}$$

The propellant tank mass, m_{tank}, is calculated to be 10 percent the propellant mass while $m_{dryprop}$ is the propulsion system dry mass. The mass of the spacecraft bus is $m_{bus}1000$ kg per our correspondence with Varda. $m_{launched}$ is the mass that Falcon 9 can launch to the current altitude. Therefore, the payload mass available for this altitude is given by Equation 6:

$$m_{payload} = m_{launched} - m_{bus} - m_{dryprop} - m_{prop} - m_{tank} \tag{6}$$

After cycling through all altitudes, drag configurations, and propulsion systems, the script produces the plots presented in the following section of this report. The script's main function is attached as Appendix A.

4 Results

4.1 MATLAB (Shubham)

The MATLAB analysis was conducted across several propulsion systems and possible altitudes to get a comprehensive view of which parameters most significantly affected the maximum payload mass that could be carried and ultimately, what the optimal altitude is for the best performing propulsion system.

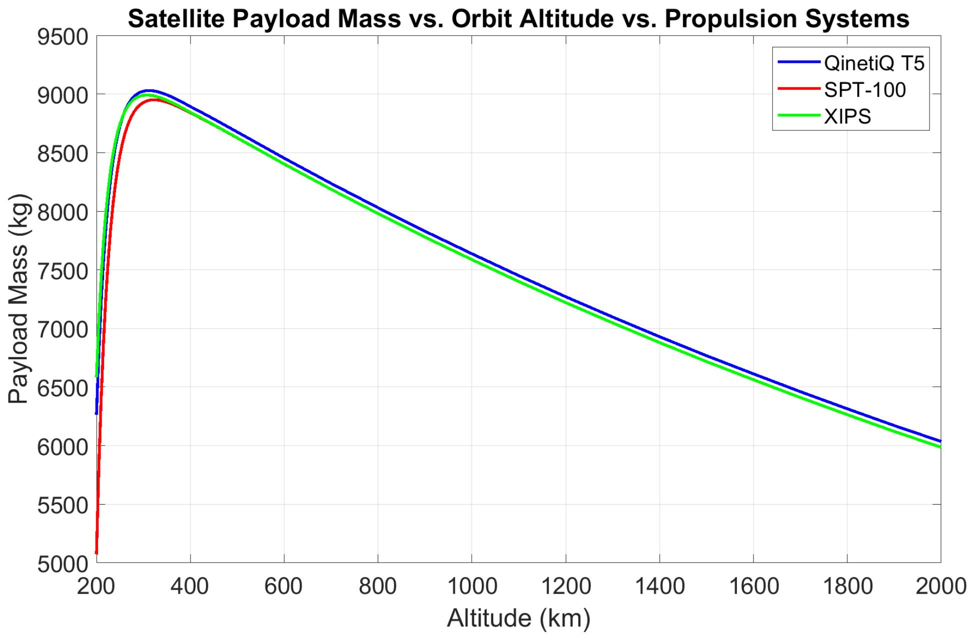


Figure 9: This graph shows the maximum payload mass for the entire range of altitudes examined

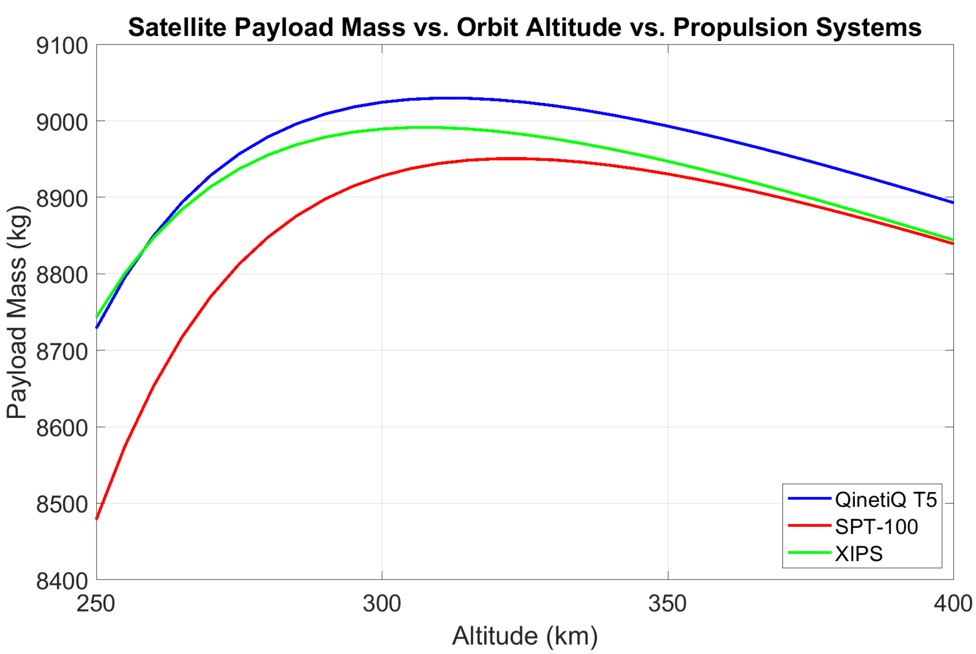


Figure 10: The graph is a zoomed in window of Figure 9 to better illustrate the optimal altitude

Shown above in Figures 9 & 10 is the maximum satellite payload mass for a range of examined altitudes for the QinetiQ T5, SPT-100, and XIPS propulsion systems, as computed by the MATLAB Script described in Section 3. All the propulsion systems follow a similar trend: At low altitudes (below 300 km), the drag forces are high and dominate the response and afterwards, is affected by how much the F9 can lift after the power system mass required to maintain that altitude. It is shown that payload mass is maximized overall with the QinetiQ T5 propulsion system at an altitude around 310 kilometers. We can conclude that the QinetiQ T5 is the best propulsion system to choose for this missions because it offers essentially the same I_{sp} as the XIPS, but is lighter. However, the percent difference in payload mass (PM) between the QinetiQ T5 and SPT-100, the best and worst propulsion systems we examined, is calculated as % difference $= \frac{|PM_{QinetiQ} - PM_{SPT}|}{PM_{QinetiQ}} = \frac{9030 - 8950kg}{9030kg} = 0.88\%$, a difference of less than 100 kg of payload.

The next step of the analysis examined the effect of varied drag due to the presence or absence of body solar cells and solar array wings. The appropriate drag coefficient, C_d, and effective area, A_{eff} for two types of body solar cells and solar array wings are applied to a rocket with the QinetiQ T5 propulsion system in Figure 11 to observe how the payload mass is affected. The body solar cells do not add to the cross sectional area, meaning A_{eff} remains the same as the Fairing F9 A_{eff}, while the solar array wings add 20 m^2 of effective area in the form of deploy-able solar panels. The two C_d values examined, 2.72 and 2.39 represent the 'no nose cone' and '0.1m tail radius' shapes, respectively, as discussed in section 2.2.

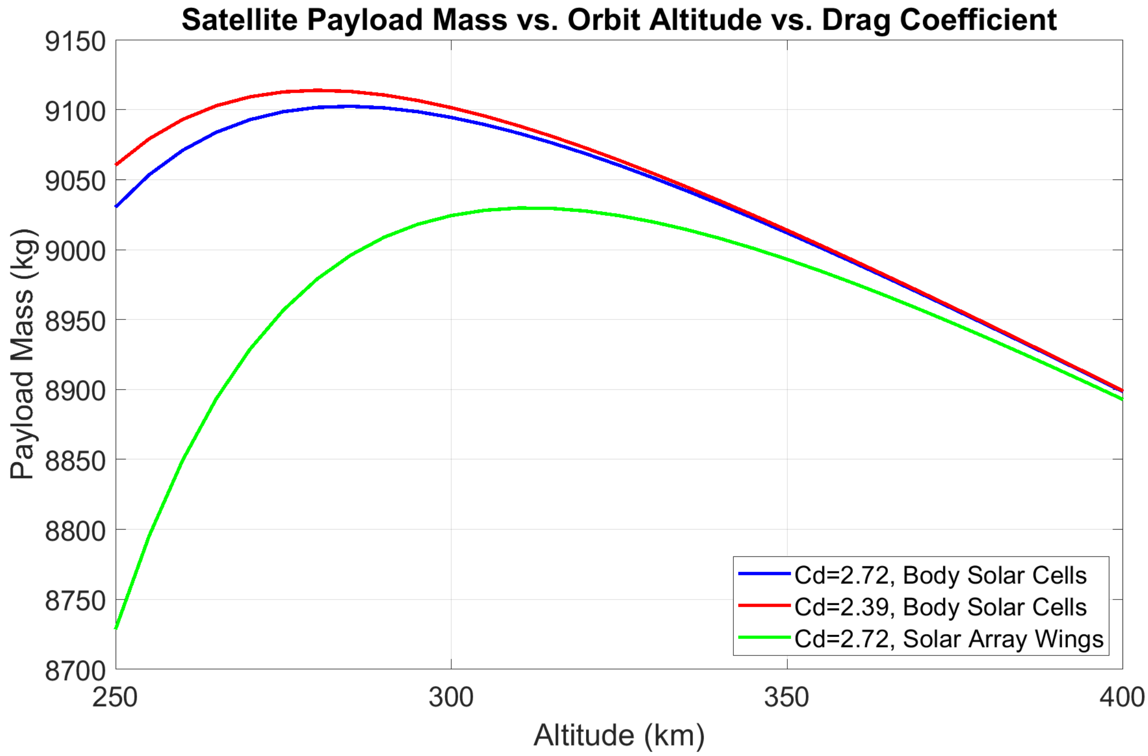


Figure 11: This figure shows the effect of varying C_d and A_{eff} on the maximum payload mass using the QinetiQ T5 propulsion system

From Figure 11, it can be seen that while the graphs are still very similar to each other, changing the shape of the rocket makes a relatively smaller difference compared to using body solar cells instead of solar array wings. The % difference in payload mass between a flat nose shape with body solar cells versus solar array wings can be calculated as % difference $= \frac{|PM_{BodyCells} - PM_{SolarArray}|}{PM_{BodyCells}} = \frac{9100 - 9030 kg}{9100} = 0.77\%$ while the difference between a flat nose and cone nose for the body solar cells is % difference $= \frac{|PM_{ConeNose} - PM_{FlatNose}|}{PM_{ConeNose}} = \frac{9110 - 9100 kg}{9110 kg} = 0.11\%$. Since the small increase in payload capacity due to a cone nose likely does not outweigh the 22% volume reduction, the best option is the flat nose tail with body solar cells.

4.2 Simulation (Mohammad)

Simulations were completed in General Mission Analysis Tool (GMAT) and compared to the results from MATLAB computations and previous studies. Assumptions and inputs that were taken in order to run the simulation included a circular orbit with an inclination angle of 28.5°due to Kennedy Space Center location on Earth [8] at a constant altitude of $290 km$. To compare to our computation results, the Jacchia-Roberts atmospheric model was used to account for density and drag perturbations on the spacecraft. The launch date of October 1^{st}, 2023, was based on Varda Space Industries' first launch mission. The orbital period of the spacecraft was determined to be 5750 seconds or ∼1.6 hours (Figure 12). This is similar to the orbital period of ISS of 1.5 hours, which is at $\sim 400km$ [9]. This agrees with the equation of orbital velocity, since the larger the semi-major axis of the orbit, the higher the orbital velocity:

$$v_{orbit} = \frac{2\pi a}{T}, \tag{7}$$

where a is the length of the semi-major axis of the spacecraft orbit, and T is the orbital period of the spacecraft (5750 seconds from simulation). Using Equation 7 the orbital velocity of the spacecraft was calculated to be $7.3km/s$ which agrees with previous results [10, 11].

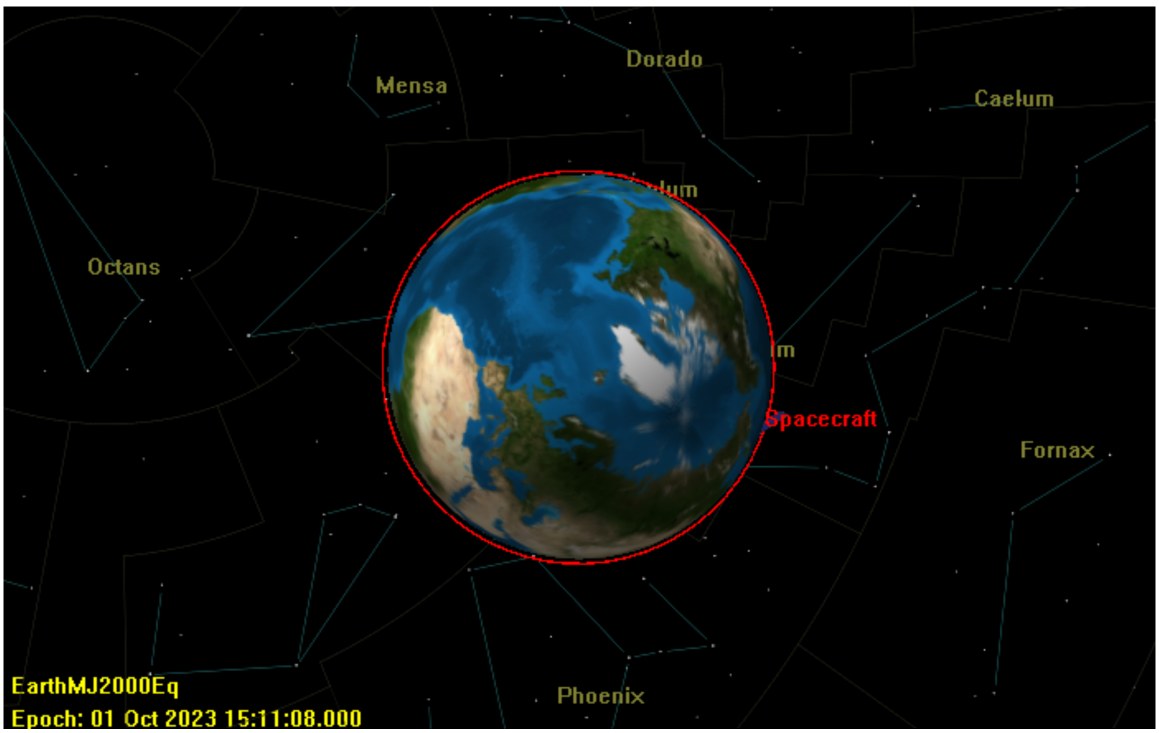
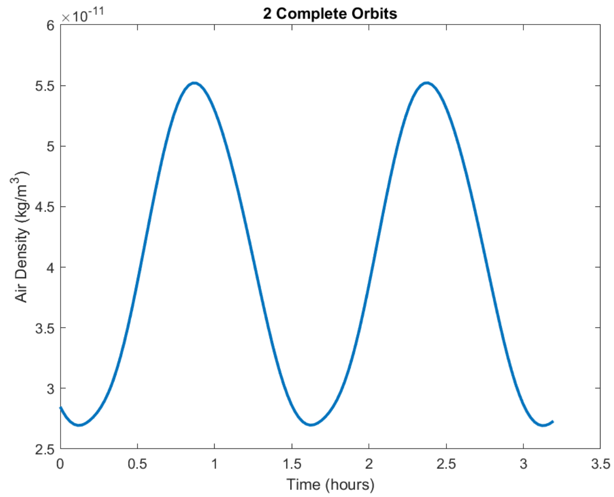
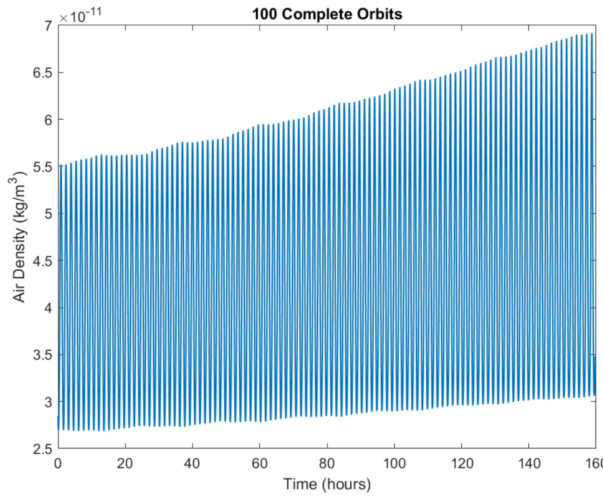


Figure 12: Simulated orbit of the spacecraft around Earth

Atmospheric density data was taken from Figure 12 simulation to generate Figure 13 plots in MATLAB. The goal of this is to verify the time invariate assumption from the MATLAB atmospheric density analysis. At an altitude of $290km$, density had a sinusoidal behavior in the order of magnitude of 10^{-11} (Figure 13a&13b). This cyclical behavior is in line with what is expected from the GRACE mission plots in Figure 1. According to the trends in the long-term run of the spacecraft in Figure 13b, it was concluded that the change in density is non-negligible. Although some of this drift is caused by a decrease in altitude (the GMAT analysis did not take into consideration the requisite variable thrust), a significant portion is likely caused by variance in solar flux. Using our custom Jacchia-Roberts script as defined in section 2.1, the density at 290 km is $1 \times 10^{-11} kg/m^3$. Therefor, our analysis is an appropriate first-order approximation that can be narrowed down using advanced simulation. The MATLAB script provides a benchmark altitude that further simulation can eventually fine-tune to find the optimal orbital radius.



(a) Behavior of density over 2 complete orbits in ~3 hours



(b) Behavior of density over 100 complete orbits in ~160 hours, or ~6.6 days

Figure 13: Plots of density as a function of time in low-Earth atmosphere

5 Conclusion (Alex and Shubham)

Mission design is an iterative process. This analysis provided a first order approximation of the optimal orbit altitude to maximize payload mass delivery for a Varda space factory in Low Earth Orbit. The factory must operate in a micro-gravity environment per the needs of Varda. This design examines the trade-off between atmospheric drag and the payload capacity of the rocket while making an informed decision on propulsion system in order to keep the factory in space for at least 5 years. The optimal configuration based on this analysis is a spacecraft with a flat nose tail, body solar cells, and launched to an altitude on the order of 300 km. This altitude optimizes the needs of the space factory (station keeping, etc.) as well as the capsule's propulsion weight requirements. Using this data, successive simulations in GMAT can provide a more precise altitude measurement.

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6 Appendix A: Altitude Optimizer MATLAB Script

```
%Clearing the workspace:
clc
clear all
close all

%Initializing constants:
g=9.8; %in m/s, gravitational acceleration
G=6.67e-11; %in m^3/kg/s^2, universal gravitational constant
m_earth=5.97e24; %in kg, the mass of the Earth
r_earth=6378e3; %in m, the radius of the Earth

altitudeArray=185:5:2000; %in km, range of altitudes we are investigating
missionLife=5*365*24*60*60; %in sec, the mission life of 5 years

%Falcon 9 payload capabilities:
orbitAlt=[linspace(200,2000,19)]; %Orbit altitude, km
launchMass= [10454 10202 9953 9727 9503 9287 9080 8879 8687 8500 ...
    8320 8147 7979 7817 7662 7510 7364 7221 7085]; %Payload mass, kg

%Drag configuration parameters:
numCds=3;
CdArray=[2.72 2.39 2.72];
AeffArray=[pi*(4.572/2)^2, pi*(4.572/2)^2, pi*(4.572/2)^2+20]; %in m^2
m_bus=1000; %in kg, the structure mass of the satellite

%Propulsion system parameters:
propSystemNames={'QinetiQ T5','SPT-100', 'XIPS'};
numPropSystems=3; %Number of prop systems we are investigating
IspArray=[3000,2200,3400]; %in sec, the Isp of the propulsion system
m_drypropArray=[50,100,100]; %in kg, the dry mass of the propulsion system

%Looping over different prop systems and Cd:
```

```
for i=1:numPropSystems

    %Setting propulsion system parameters:
    Isp=IspArray(i);
    m_dryprop=m_drypropArray(i);

    %Looping over different Cds:
    for j=1:numCds

        Cd=CdArray(j);
        Aeff=AeffArray(j);

        %Looping over different altitudes:
        for k=1:length(altitudeArray)

            altitude=altitudeArray(k); %in km, altitude
            rho(k)=calcAirDensity(altitude); %in kg/m^3

            %in kg, the maximum mass a Falcon 9 can lift to this altitude:
            m_launched=interp1(orbitAlt,launchMass,altitude,'linear');
            m_launchedArray(i,j,k)=m_launched;

            v=sqrt(G*m_earth/(r_earth+altitude)); %in m/s, circular orbit
            F_drag=0.5*Cd*Aeff*rho(k)*v^2; %in N, drag force
            F_thrust=F_drag; %in N, thrust force
            mdot=F_thrust/(Isp*g); %Calculate mdot for the prop system

            m_prop=missionLife*mdot; %Propellant mass
            m_tank=0.1*m_prop; %Assume Tank mass is 10% of propellant mass
            m_payload=m_launched-m_bus-m_dryprop-m_prop-m_tank;

            %Storing the calculated payload mass:
            m_payloadMatrix(i,j,k)=m_payload;
            F_dragArray(i,j,k)=F_drag;

        end
```

```
    end

end

%Creating plots:
figure(1)
hold on
grid on
box on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
plot(altitudeArray,squeeze(m_payloadMatrix(1,3,:)),'b','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(2,3,:)),'r','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(3,3,:)),'g','LineWidth',2);
title('Satellite Payload Mass vs. Orbit Altitude vs. Propulsion Systems')
legend(propSystemNames{1},propSystemNames{2},propSystemNames{3},...
    'Location','northeast');
xlabel('Altitude (km)');
ylabel('Payload Mass (kg)');
xlim([200 2000]);
set(gca,'FontSize',16)

figure(10)
hold on
grid on
box on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
plot(altitudeArray,squeeze(m_payloadMatrix(1,3,:)),'b','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(2,3,:)),'r','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(3,3,:)),'g','LineWidth',2);
title('Satellite Payload Mass vs. Orbit Altitude vs. Propulsion Systems')
```

```matlab
legend(propSystemNames{1},propSystemNames{2},propSystemNames{3},...
    'Location','southeast');
xlabel('Altitude (km)');
ylabel('Payload Mass (kg)');
xlim([250 400]);
set(gca,'FontSize',16)

figure(2)
hold on
grid on
box on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
plot(altitudeArray,squeeze(m_payloadMatrix(1,1,:)),'b','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(1,2,:)),'r','LineWidth',2);
plot(altitudeArray,squeeze(m_payloadMatrix(1,3,:)),'g','LineWidth',2);
title('Satellite Payload Mass vs. Orbit Altitude vs. Drag Coefficient')
legend('Cd=2.72, Body Solar Cells','Cd=2.39, Body Solar Cells',...
    'Cd=2.72, Solar Array Wings','Location','southeast');
xlabel('Altitude (km)');
ylabel('Payload Mass (kg)');
xlim([250 400]);
set(gca,'FontSize',16)

figure(3)
semilogy(altitudeArray,squeeze(F_dragArray(1,1,:))*1000,'b','LineWidth',2);
title('Drag Force vs. Orbit Altitude')
xlabel('Altitude (km)');
ylabel('Drag Force (mN)');
grid on
box on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
set(gca,'FontSize',16)

```

```
figure(4)
hold on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
plot(altitudeArray,squeeze(m_launchedArray(1,1,:)),'-b','LineWidth',2);
plot(orbitAlt,launchMass,'.b');
title('Falcon 9 Payload Mass vs. Insertion Altitude')
xlabel('Altitude (km)');
ylabel('Mass (kg)');
grid on
box on
set(gca,'FontSize',16)

figure(5)
semilogy(altitudeArray,rho,'b','LineWidth',2);
title('Air Density vs. Altitude')
xlabel('Altitude (km)');
ylabel('Air Density (kg/m^3)');
grid on
box on
set(gcf, 'Position',[0 0 1000 1000])
pbaspect([1.6 1 1])
set(gca,'FontSize',16)
```